Lecture 27 - Thermal conduction, Radiation

A heated object may transfer energy to its surroundings by a variety of processes, including: Thermal conduction, convection(fluids) and radiation. Thermal conduction occurs in all materials. In fluids, that is liquids and gases, convection may occur, and if it does, it is usually dominant. Heat is always also lost through radiation, and this process is dominant for hot bodies in dilute gases or in vaccuum and occurs by the emission of photons. Here, we consider only thermal conduction and radiation.

Thermal conduction

Thermal conduction through a piece of material is characterized by a material property called the thermal conductivity, k, so that the heat flow due to a temperature difference ΔT is given by,

$$\frac{Q}{\Delta t} = \frac{kA}{L}\Delta T \tag{1}$$

where A is the cross-section of the piece of material, L is its length and ΔT is the difference in temperature between its two ends. The heat flow is from the hot side of the piece of material to the cool side.

If two thermal barriers are combined in series, their total thermal barrier is increased. Thermal insulation in a house is an example of a heat barrier which uses low thermal conductivity materials. Double glazed windows are a combination of two sheets of glass with an air gap between them. The overall effect of a series combination like this is treated using the following formula

$$\frac{Q}{\Delta t} = A\frac{\Delta T}{\sum R_i} \tag{2}$$

where the R factor of the insulation is given by,

$$R_i = \frac{L_i}{k_i} \tag{3}$$

For $i = 1$, this is the same as Eq. (1). A high R factor can be attained by using either a thick layer of insulation or by using materials with a very low value of their thermal conductivity.

However, in some applications you want very high thermal conductivity, for example in microelectronics as the heat losses need to be transported away from transistors etc. Diamond has the highest thermal conductivity, while vaccuum has the lowest. Gases such as air also have low thermal conductivity and it gets lower as the density of the gas decreases. However if convection occurs, heat transport occurs more rapidly in a gas, so to get good thermal insulation it is necessary to make porous materials where the air gaps are designed to prevent convection. In micro-electronics, the problem is the opposite. Transistors produce heat and this heat must be removed or the circuits will fail. The next generation of integration in electronics faces severe problems with the relatively low thermal conductivity of silicon, which is now a key factor limiting further miniturization of circuitry. There is thus a push toward diamond-like materials as microelectronic substrates in order to increase their thermal conductivity.

Example: What is the rate at which heat is lost when the outside temperature is 10^0F and the inside temperature is 70^0F through (i) The windows of a house with total area $A = 40m^2$ and $R = 3$ (good windows and drapes); (ii) The roof and walls of a house with total area $A = 400m^2$ and $R = 20$ (good insulation).

Solution The temperature difference must be calculated in kelvin, so we have $\Delta T = 60 * 5/9K$. The rate of heat loss (power loss) through the windows is then, $P = 40 * 300/27\ watts = 444W$. The rate of heat loss (power loss) though the roof and walls is, $P = 400 * 300/(9 * 20) watts = 666W$. The typical energy cost is about $8 cents/kWhr$. The heat loss in this house is $666 + 444 = 1.1kW$, so the cost of heating the house is approximately $24 * 0.08 * 1.1$ which is roughly two dollars a day. This calculation does not take into account serious heat leaks, like open windows, drafty ceilings, bad storm doors etc, which can significantly add to the cost.

Radiated heat

A hot body emits photons which carry away energy from the source, this is radiated heat. The rate at which heat is radiated by a body of surface area A, at temperature T is given by the Stefan-Boltzmann law,

$$P = \sigma e A T^4 \tag{4}$$

If this body is at equilibrium with a surrounding where the surrounding is at temperature T_0, then the power emitted becomes,

$$P = \sigma e A(T^4 - T_0^4) \tag{5}$$

In these expressions e is the emissivity of the body and varies from $e = 1$ which corresponds to an ideal absorber or "black body", while $e = 0$ is the limit of a perfect reflector. $\sigma = 5.67 * 10^{-8} W/m^2K^4$ is the Stefan-Boltzmann constant.

Example. The sun has a surface temperature of $T = 6000K$. Assuming that the sun is a perfect blackbody (e=1) and given that the radius of the sun is $R_s = 696,000km$ and that the earth-sun distance is $R_{es} = 150$ *million km*, and that the radius of the earth is $R_e = 6400km$, estimate the surface temperature of the earth for $e = 0.5, e = 0.75, e = 1.0$.
Solution. Assume that the amount of energy absorbed by the earth is equal to that emitted. The amount of energy absorbed by the earth from the sun is given by,

$$P_s = (\pi R_e^2)(\sigma T_s^4)\frac{4\pi R_s^2}{4\pi R_{es}^2} = (\pi R_e^2)\frac{\sigma T_s^4 R_s^2}{R_{es}^2} \tag{6}$$

Note that the area used in the first factor on the RHS is πR_e^2 NOT $4\pi R_e^2$ as the former is the cross section seen by the incident sunlight. The amount of energy emitted by the earth is given by

$$P_e = \sigma 4\pi R_e^2 e T_e^4 \tag{7}$$

At steady state, the incident power and the emitted power are the same, so we equate $P_e = P_s$ to find that

$$T_e = (\frac{a}{4e})^{1/4} T_s (\frac{R_e}{R_{es}})^{1/2} \tag{8}$$

Using $e = 1$, we get $T_e = 289K$, which is about right, if we use $e = 1/2$, we get $T_e = 344K$. This shows that small changes in the emissivity, e, of earth's atmosphere can lead to very significant changes in the surface temperature of the earth. Absorbing gases, like CO_2, are decreasing e and have the potential to raise the surface temperature of the earth, however we need much better models to estimate by how much as there are many ways in which the oceans and atmosphere may respond to the increased energy stored in the ecosystem. Venus is an example of a planet where the greenhouse effect has gone wild. Venus is 108 *million km* from the sun and it has a mass and hence gravitational acceleration which is slightly smaller than earth's. However its surface temperature is $482^0C = 775K$ and its surface pressure is 90 times that at the surface of the earth. Almost all of the water and carbon dioxide on Venus is in its atmosphere and this produces a really large greenhouse effect so that a massive amount of energy is stored in its atmosphere. If we could take that energy out of the atmosphere of Venus, its average surface temperature would be close to that of earth's. If fact using the distance $108,000,000$ km and an emissivity of one, we can find the expected surface temperature of Venus.